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Application of Sliding Mode Control with Extended High Gain Observer to Stabilize the Underactuated Quadrotor System

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ABSTRACT

This work proposes an output feedback controller for stabilization of the quadrotor underactuated system in the presence of time varying disturbances and model uncertainties. The proposed control is an improvement to the sliding mode control (SMC). An extended high-gain observer (EHGO) when combined with sliding mode control (SMC) able to give feasible performance beyond the performance of the standard sliding mode. It is able to bring the state trajectories of the closed-loop system close to the target system with a smaller ultimate bound of error and smaller control magnitude. The proposed method is illustrated by simulation.

Keywords: Extended high-gain observer, sliding mode control, underactuated system, output feedback control

INTRODUCTION

The vertical take-off and landing (VTOL) vehicle such as quadrotor is perceived to have good potential in various applications such as monitoring, surveillance, and search and rescue (SAR) mission. The quadrotor hovering capability makes it the best choice for near monitoring applications within confined areas. The quadrotor unmanned aerial vehicle (UAV) is in the group of underactuated system because of its four-input actuator that allows to control six degree of freedom outputs. It is classified as a second order nonholonomic, thus the controller design and stability analysis are complicated. The control problem of quadrotors has been confronted using several different approaches from leading research teams worldwide. Earlier works focuses on stabilization of the vehicle using linear approach (Bouabdallah

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et al., 2004) and nonlinear approach (Hoffmann et al., 2007; Bouabdallah & Siegwart, 2007; Benallegue et al., 2006).However, the stability is not guaranteed when the vehicle is flying in the presence of model uncertainties and external disturbances.

The controller design that includes uncertainties and disturbances and capable of disturbance rejection generally focuses on two direction, either adaptive method such as the work by Chen et al. (2014) or disturbance-observer (DOB) method. DOB has advantage as opposed to the adaptive technique in terms of flexibility and design simplicity (Dong et al., 2014). In DOB-based controller, the nominal model is retained while an observer is designed and added into the control to estimate and cancel the disturbance.

The study on robust control for trajectory tracking in real time is still new. A robust sliding-mode or high-gain observer can be used as the DOB-based controller to perform this mission. The use of sliding-mode observer had been reported by several authors (Benallegue et.al., 2008; Besnard et al., 2012). However, the use of high-gain observer as DOB-based for robust trajectory control of a quadrotor is still lacking so far. According to (Freidovich & Khalil, 2008) the high-gain observer is simpler compared to the sliding-mode approach.

The backstepping technique combined with DOB for robust trajectory tracking of the quadrotor proposed by (Dong et al., 2014) is promising. However, the backstepping method is not suitable for a complex system because the controlling algorithm is based on a recursive method involving complex mathematics. In complex mission, the backstepping controller will involve heavy mathematical coding and computation which is time consuming and error prone during start up.

Sliding mode control is one of the well-known approaches for handling nonlinear systems that are under presence of uncertainties and external disturbances (Mokhtari & Cherki, 2015). The advantage of sliding mode control lies in its robustness and simplicity of implementation. However, the drawback of the sliding mode control is the chattering effect arising from high frequency switching. The chattering effect is usually solved by replacing the discontinuous switching to continuous switching. However, the drawback of using continuous switching is the error convergence of the states is uniformly ultimately bounded, instead of converging to zero in finite time.

A controller based on sliding mode control proposed by Xu and Özgüner (2008) for stabilizing a class of underactuated systems has an attractive sliding surface. The sliding surface presented is able to globally stabilize all degrees of freedom including those which are indirectly actuated through the nonlinear coupling. However, the ultimate bound of the steady state error it produces is large. In this paper, we propose an improvement to the controller proposed by (Xu & Özgüner, 2008) . We use an extended high-gain observer (EHGO) as estimator to estimate the unknown states and the uncertainties and disturbances. The estimated states will be used in the controller and at the same time the estimated uncertainties and disturbances are continuously cancelled in the control. In simulation, we are able to show that our proposed method able to improve the performance of the standard sliding mode controller. The proposed control able to give smaller ultimate bound of error at a smaller magnitude of control signal.

This paper begins with a presentation of the dynamic model of the quadrotor vehicle and the transformation of the model to a cascade form for control design. This is followed with a presentation of proposed controller in a state feedback form. We assume that all states, uncertainties and disturbances are known and available for the controller. An analysis between the proposed controller and standard SMC approach is undertaken and the proposed design of EHGO and output feedback control is presented. The efficiency of the proposed controller is illustrated through simulation using Matlab and Simulink.

PRELIMINARIES

Dynamic Model and Transformation

A simplified nominal model of a quadrotor UAV as shown in Figure 1 can be represented as follows. More detail of its configuration can be found in (Bouabdallah, 2007;Altug, Ostrowski, & Mahony, 2002)

$$\left.\begin{array}{c}\begin{bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\end{bmatrix} = U_1\begin{bmatrix}\cos\emptyset\sin\theta\cos\varphi+\sin\emptyset\sin\varphi\\ \sin\emptyset\sin\theta\cos\varphi-\cos\emptyset\sin\varphi\\ \cos\emptyset\cos\varphi\end{bmatrix}-\begin{bmatrix}\frac{K_1\dot{x}}{m}\\ \frac{K_2\dot{y}}{m}\\ \frac{K_3\dot{z}}{m}\end{bmatrix}-\begin{bmatrix}0\\0\\1\end{bmatrix}g\\ \begin{bmatrix}\ddot{\theta}\\ \ddot{\phi}\\ \ddot{\psi}\end{bmatrix}=\begin{bmatrix}U_2\\U_3\\U_4\end{bmatrix}-\begin{bmatrix}lK_4\dot{\theta}/I_1\\ lK_5\dot{\varphi}/I_2\\ K_6\dot{\emptyset}/I_3\end{bmatrix}\end{array}\right\} \quad (1)$$

where $[x, y, z]^T$ are the position in the x -axis, y -axis and z -axis ; and $[\theta, \phi, \psi]^T$ are the pitch, roll and yaw, respectively; g is the acceleration of gravity, l is the half length of the helicopter, m is the total mass of the helicopter; I_i, K^i, U_i, (i = 1, 2 ,3 are the moment of inertia with respect

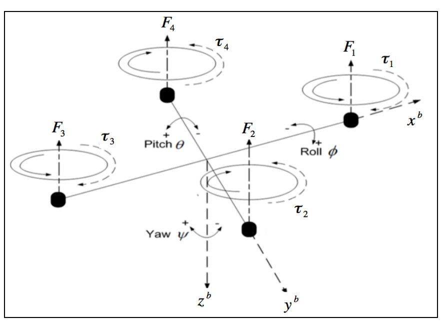

Figure 1. Quadrotor configuration

to the axes, the drag coefficients, and the control inputs, respectively. ψ

The quadrotor system can be divided into two subsystems; a fully actuated and an underactuated. The quadrotor height (z) and yaw motion are assumed to be fully actuated. Meanwhile, the positions in the longitudinal (x) and lateral (y) and the pitching and rolling angle are a multi-input multi-output (MIMO) underactuated subsystem represented as

$$\left.\begin{array}{c}\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} = U_{1_1}\begin{bmatrix}\cos\emptyset & \sin\emptyset\\ \sin\emptyset & -\cos\emptyset\end{bmatrix}\begin{bmatrix}\sin\theta\cos\varphi\\ \sin\varphi\end{bmatrix}+\frac{1}{m}\begin{bmatrix}-K_1\dot{x}\\ -K_2\dot{y}\end{bmatrix}\\ \begin{bmatrix}\ddot{\theta}\\ \ddot{\varphi}\end{bmatrix}=\begin{bmatrix}U_2\\U_3\end{bmatrix}+\begin{bmatrix}-lK_4\dot{\theta}/I_1\\ -lK_5\dot{\varphi}/I_2\end{bmatrix}\end{array}\right\} \quad (2)$$

The controller design for the fully actuated subsystem is constructed using sliding mode and PID-based control following (Xu & Özgüner, 2008). In this paper, we will focus on the design of disturbance rejection mechanism to optimize the control performance for the underactuated subsystem Eq. (2) in the presence of time-varying disturbances and uncertainties. The control of underactuated system is important for stabilizing the vehicle in the longitudinal and lateral motion during trajectory.

We define new state variable as $x_1 = \begin{bmatrix} x \\ y \end{bmatrix}, x_2 = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}, x_3 = \begin{bmatrix} \theta \\ \varphi \end{bmatrix}, x_4 = \begin{bmatrix} \dot{\theta} \\ \dot{\varphi} \end{bmatrix}$ to obtain the transformation of system Eq. (2) into

$$\left.\begin{aligned} \dot{x}_1 &= x_2 + d_1 \\ \dot{x}_2 &= f_1(x_3) + d_2 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= U + d_3 \end{aligned}\right\} \tag{3}$$

where x_1 and x_2 are the position and angular position, respectively, x_2 and x_4 are the position and angular velocity, respectively, $U \in R^2, U = \begin{bmatrix} U_2 \\ U_3 \end{bmatrix}, f_1 = \begin{bmatrix} sin\theta cos\varphi \\ sin\varphi \end{bmatrix}, d_1 \in R^2, d_1 = \frac{\dot{u}_1}{u_1} x_1,$ d_2 and d_3 are a lumped of disturbances and uncertainties appearing in the translational link (x_1 and x_2) and rotational link (x_3 and x_4), respectively, $d_2 = \begin{bmatrix} \frac{K_1 \dot{x}}{m} + 0.03\sin(0.5t) \\ \frac{K_2 \dot{y}}{m} + 0.03\sin(0.5t) \end{bmatrix},$ $d_3 = \begin{bmatrix} \frac{lK_4 \dot{\theta}}{I_1} + 0.003\sin(0.5t) \\ \frac{lK_5 \dot{\varphi}}{I_2} + 0.003\sin(0.5t) \end{bmatrix},$ and x_1 and x_2 are available for measurement. Assumptions are made as follows:

Assumption 1: d_1, d_2, and d_3 belongs to a compact set and are bounded. $\dot{d}_2$ and $\dot{d}_2$ are bounded.
Assumption 2: $\frac{df_1}{dx_3}$ is invertible, $\frac{df_1}{dx_3}$ and $b(\bullet)$ are continuously differentiable with locally Lispchitz derivatives.
We define the error variable $e_1 = x_1 - x_{1d}$, $e_2 = x_2$, $e_3 = f_1$, and $e_4 = \frac{df_1}{dx_3}$, , the switching surface

$$s = c_1 e_1 + c_2 e_2 + c_3 e_3 + e_4 \tag{4}$$

is defined. The dynamics of the switching surface

$$\dot{s} = f_k + f_d + gU \tag{5}$$

where $f_k = c_1 x_2 + c_2 f_1 + c_3 \left[\frac{\mathrm{df1}}{\mathrm{dx3}}\right] x_4 + \frac{d}{dt}\left[\frac{\mathrm{df1}}{\mathrm{dx3}}\right] x_4, f_d = c_1 d_1 + c_2 d_2 + d_3$ and $g = \frac{\mathrm{df1}}{\mathrm{dx3}}$. . The control goal is to design the control U that is able to bring the sliding surface, s in Eq. (4) to zero in finite time under presence of d_2 and d_3. This will result in improved performance of position trajectory specifically, smaller ultimate bound of position error, e_1 with lesser magnitude of control signal.

CONTROL DESIGN

Standard Sliding Mode Control

We first present the standard sliding mode control algorithm for the underactuated system proposed by (Xu & Özgüner, 2008). The sliding mode control and the closed-loop control of the sliding surface are represented as Eqs. (6) and (7), respectively, as following.

$$U = g^{-1}\left(-f_k - M_1 sat\left(\frac{s}{\mu}\right) - \lambda s\right) \tag{6}$$

$$\dot{s} = -M_1 sat\left(\frac{s}{\mu}\right) - \lambda s + d \tag{7}$$

The closed-loop control of the sliding surface is obtained by combining Eq. (5) - (6). d represents the lumped of d_2 and d_3 , and M_1 is the saturation level which is determined based on the upper bound of d.

Proposed EHGO based Sliding Mode Control

We proposed an observer to estimate d_2 and d_3 and then continuously cancelling the estimated terms in the sliding mode control. The control algorithm of the proposed technique and the closed-loop control of the sliding surface are represented as in Eqs. (8) and (9), respectively.

$$U = g^{-1}\left(-f_k - f_{\hat{d}} - M_1 sat\left(\frac{s}{\mu}\right) - \lambda s\right) \tag{8}$$

$$\dot{s} = -M_1 sat\left(\frac{s}{\mu}\right) - \lambda s \tag{9}$$

where $f_{\hat{d}}$ is the estimated disturbance which will be generated by the EHGO in the next subsection. In our proposed method, M_1 can be any value smaller than the value chosen in standard SMC Eq. (6). M_1 does not need to depend on the upper bound of the disturbance and uncertainties because it is cancelled in the control.

Analysis of Ultimate Bound of |s|

The sliding surface trajectory of Eq. (4) is analysed. The performance of the sliding surface outside the boundary layer$\{|s| \geq \mu\}$ is the same using the proposed method Eq. (8) and using the standard SMC Eq. (6). Both controllers are able to bring the sliding surface to the boundary layer in finite time.

However, inside the boundary layer $\{|s| \leq \mu\}$, $s\dot{s} \leq -\left(\frac{M_1}{\mu} + \lambda\right)s + s|a|$, $a \gg |d|$. In the standard SMC, the disturbance d is dominated. Therefore, the ultimate bound of |s| will be equal to $\frac{a}{\frac{M_1}{\mu}+\lambda}$, , where the bound will depend on the size of α In contrast, the proposed control Eq. (8) results in ultimate bound of |s| to be equal to $\frac{0}{\frac{M}{\mu}+\lambda}$, due to cancellation of the disturbance. Theoretically, the error bound of $|s|$ is zero for proposed control.

Design of EHGO and Output Feedback Control

The EHGOs for the position and rotational dynamics are designed as two different observer (Khalil, 2014):

$$\left.\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 - \left(\frac{\dot{U}_1}{U_1}\right)\hat{x}_1 + \frac{\alpha_{11}}{\varepsilon}(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= f_1(x_3) - \left(\frac{\dot{U}_1}{U_1}\right)\hat{x}_2 + \hat{d}_2 + \frac{\alpha_{12}}{\varepsilon^2}(x_1 - \hat{x}_1) \\ \dot{\hat{d}}_2 &= \frac{\alpha_{13}}{\varepsilon^3}(x_1 - \hat{x}) \end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned} \dot{\hat{x}}_3 &= \hat{x}_4 + \frac{\alpha_{21}}{\varepsilon}(x_3 - \hat{x}_3) \\ \dot{\hat{x}}_4 &= \hat{d}_3 + u + \frac{\alpha_{22}}{\varepsilon^2}(x_3 - \hat{x}_3) \\ \dot{\hat{d}}_3 &= \frac{\alpha_{23}}{\varepsilon^3}(x_3 - \hat{x}_3) \end{aligned}\right\} \tag{11}$$

Eq. (10) is the EHGO for position and Eq. (11) is the EHGO for rotational subsystem, $\hat{d}_2$ and $\hat{d}_3$ denote the estimate of disturbance and uncertainties in the translational and rotational link, respectively. The constants α_{ji} for $i = 1,2$ and $j = 1,2,3$ are chosen such that the following polynomials $s^3 + \alpha_{i1}s^2 + \alpha_{i1}s + \alpha_{i3}$, for $i = 1,2$ are Hurwitz and ε is a small positive number. Combining Eqs. (8), (10) -(11), the output feedback control is obtained as follows:

$$U = -g^{-1}\{-f_k - f_{\hat{d}} - M_1 sat\left(\frac{\hat{s}}{\mu}\right) - \lambda\hat{s}\} \tag{12}$$

Due to the high-gain term, ε in the EHGO, peaking will occur before the transient response. The smaller the ε, the higher the peaking (Khalil & Praly, 2014). The peaking destabilizes the control system. Therefore, to protect the system from peaking we saturate the control outside compact set of interest. The saturation function *sat*(•) is used. Saturating the expression of U at $\pm M_2$ using *sat*(•) we arrived at the output feedback controller

$$U = M_2 sat\left(\frac{-g^{-1}\{-f_k - f_{\hat{d}} - M_1 sat\left(\frac{\hat{s}}{\mu}\right) - \lambda\hat{s}\}}{M_2}\right) \tag{13}$$

The saturation value M_2 is determined such that the saturation functions will not be invoked under state feedback.

Theorem 1: Consider the closed-loop system formed of the plant Eq. (3), the observer Eqs. (10) -(11) and the controller Eq. (13). Suppose Assumptions 1-2 are satisfied. The initial states of the observer belong to a compact subset of R^{n+1}, and the initial states of the system belong to a compact set interior of Ω_c . Then, there exists $\bar{\varepsilon} > 0$ such that for $\varepsilon \in (0, \bar{\varepsilon})$: :

- all trajectories are bounded;
- $\|x(t) - x^*(t)\| \to 0 \; as \; \varepsilon \to 0$, uniformly in $t, t \geq 0$;
- $\|x(t)\|$ is uniformly ultimately bounded by $\delta(\varepsilon)$, where $\delta(\varepsilon) \to 0$ as $\varepsilon \to 0$

Proof of Theorem 1

A singular perturbation method is used to analyse the EHGO. First, the error between the actual states and estimated states are defined as follows

$$\eta_{1x} = \frac{x_1 - \hat{x}_1}{\varepsilon^2}, \quad \eta_{2x} = \frac{x_2 - \hat{x}_2}{\varepsilon}, \quad \eta_{3x} = d_2 - \hat{d}_2. \tag{14}$$

Differentiating η_{1x}, η_{2x}, η_{3x} in Eq. (14) and substituting into Eq. (3), the following matrix is obtained:

$$\varepsilon\dot{\eta}_{ix} = A\eta_{ix} + \varepsilon\Delta_1\eta_{ix} + \Delta_2\dot{d}_2, \quad i = 1,2,3 \tag{15}$$

where $A = \begin{bmatrix} -\alpha_1 & 1 & 0 \\ -\alpha_2 & 0 & 1 \\ -\alpha_3 & 0 & 0 \end{bmatrix}$, $\Delta_1 = \begin{bmatrix} -\left(\frac{\dot{U}_1}{U_1}\right) & 0 & 0 \\ 0 & -\left(\frac{\dot{U}_1}{U_1}\right) & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\Delta_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.. If $\frac{\dot{U}_1}{U_1}$ and $\dot{d}_2$ are bounded, then after short period of time, $\dot{\eta}_{ix} = 0(\varepsilon)$, for i = 1,2,3.

SIMULATION

The simulation is done using Matlab SIMULINK. The quadrotor model parameters are: $m = 2kg, l = 0.2m, g = 9.8\text{m/s}^2, I_1 = I_2 = 1.25\ Ns^2/rad, I_3 = 2.5\ Ns^2/rad, K_1 = K_2, = K_3 = 0.010\ Ns/m$, $K_4 = K_5, = K_6 = 0.012\ Ns/rad$. The state feedback controller described by Eq. (6) was implemented using the following parameter values: c_1=20, c_2=22,c_3=8, M_1=6, μ=0.1, η=0.1.

The proposed output feedback controller which is described by Eq. (13) was implemented using the same parameter values as the state feedback mentioned above. The saturation limits are chosen to be slightly greater than the maximum absolute values of the states, respectively, observed in state feedback control simulations. Meanwhile, the following parameter value were used for EHGO Eq. (10) -(11): $\alpha_{11} = \alpha_{12} = \alpha_{21} = \alpha_{22} = 3$, and $\alpha_{13} = \alpha_{23} = 1$. The initial states of state feedback and output feedback are $x_1(0) = 2$, 2,$x_2(0) = 0$,$x_3(0) = 0.5$, and $x_4(0) = 0$. The initial conditions set for EHGO are $\hat{x}_1(0) = 0.1$, $\hat{x}_2(0) = 0.1, \hat{x}_3(0) = 0, \hat{x}_4(0) = 0, \hat{d}_2(0) = 0$ and $\hat{d}_3(0) = 0$..

To investigate the performance of the proposed control with regard to the standard SMC, we simulate the closed-loop system using proposed control Eq.(13) at three cases : EHGO at ε = 0.01, ε = 0.002 and ε = 0.001.The results are shown in Figure 2 and 3. Although the transient response trajectories as shown in Figures (2a)(2c) and Figures (3a)(3c) are showing large deviation in the overshoot and settling time as compared to the standard SMC, however as ε reduces, the overshoot and settling time improves slightly. It is expected that the proposed controller to produce slight deviation in the transient response from the standard SMC. This is because the proposed control is an output feedback form while it is compared to the standard SMC that is in a state feedback form. In the standard SMC Eq. (6), we assumed all states are known and available to be used in the control. However, this assumption is not valid in

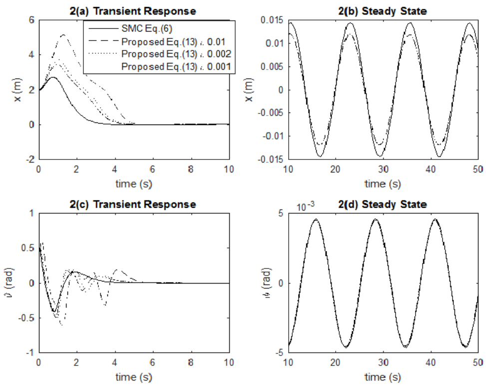

Figure 2. Trajectories of x and θ

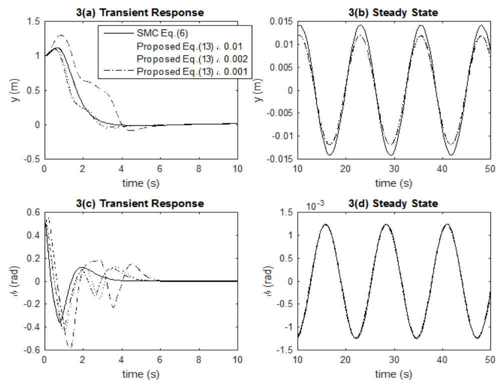

Figure 3. Trajectories of y and ψ

practical settings due to limitation of sensors. Contrary to that, the proposed method is more practical because it assumes limited sensors are available and uses robust observer to estimate other unknown states.

The efficiency of our proposal is obvious at the steady state as shown in Figure 2(b) and 2(d) and Figure 3(b) and 3(d). The proposed technique and standard SMC able to bring the trajectory of x and y to converge to ultimate bound around zero. However, inside the boundary layer the proposed technique able to bring the trajectory to smaller ultimate bound which means closer to the desired position as shown in Figure 2(b) and Figure 3(b). The performance of the proposed method also depends on the gain α_1, α_2, α_3 from the EHGO. As the gains increases as shown in Figure 4, the overshoot and the settling time are getting smaller and the transient response is closely following the standard SMC. The proposed technique can be implemented at smaller control magnitude as presented in Figure 5. The result justifies t using disturbance estimator for the purpose of estimating the disturbance and then cancel it in the control gives smaller ultimate bound in the position trajectory with smaller control effort needed to produce that performance, as compared to dominating the disturbance.

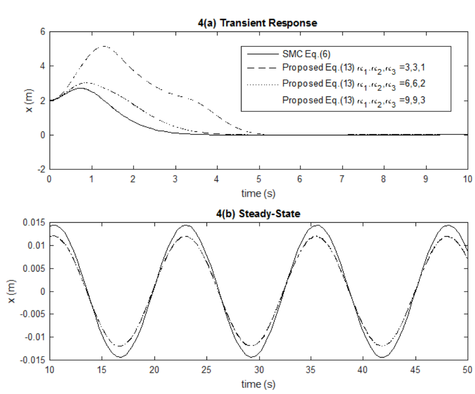

Figure 4. x trajectories at 0.001 and varying α_1, α_2, α_3

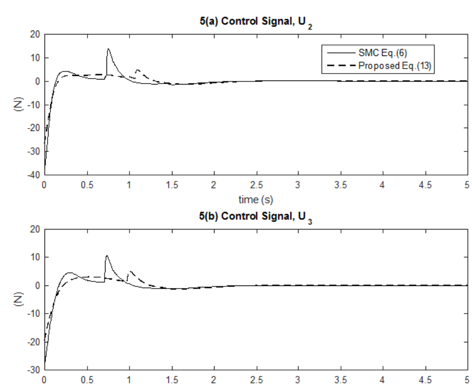

Figure 5. Control signal

CONCLUSION

We presented a more robust output feedback controller for stabilization of the under-actuated part of the quadrotor system which is continuous and time-varying. An EHGO is used to estimate the unmeasured states and to compensate for the disturbances and uncertainties that appear in the positional and rotational link of the quadrotor. The efficiency of the proposed technique was compared over the standard sliding mode control. Numerical simulation carried out shows that the proposed output feedback control produces the same output response as the state feedback sliding mode control, with the exception of some short overshoot and higher settling time. However, the proposed output feedback is able to improve the steady state error utilizing smaller magnitude of control signal.

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